

ECON-7102: Environmental Economics I

Class 11: Revealed preference models

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Today's agenda

1. Questions?
2. Housekeeping:
 - Abstract 3 due Sun Oct 11, 11:59 PM (Canvas). Pitches Mon Oct 12.
3. Today:
 - Revealed preference models for valuing the environment
4. For next class...

Starting point

- **Exact welfare measures for changes in the quantity/quality of an environmental good cannot be obtained using ordinary techniques**
 - Compensated demand cannot be estimated/observed (in general)
 - Ordinary demand cannot be estimated for environmental goods (not traded in markets).
- In the previous lecture, we arrived at the following “Slutsky-style” relationship between ordinary/compensated demand for quality changes:

$$\pi^q(p, u^0, q) = -\frac{\partial E(p, u^0, q)}{\partial q} \equiv \frac{\partial V(p, y, q)/\partial q}{\partial V(p, y, q)/\partial y} = \theta^q(p, y, q),$$

but unlike with price changes, θ^q is not estimable with market data.

- **We can impose more structure/assumptions about the relationship between environmental goods (q) and market goods (x) to measure welfare.**
 1. Complements
 2. Substitutes
 3. Quality-differentiated market goods

Willingness to pay & revealed preference

- We want to know what individuals are willing to pay for environmental quality.

$$WTP = \int_{q^0}^{q^1} \pi^q(p, u^0, q) dq = y - E(p, u^0, q^1)$$

- We could ask individuals their WTP directly
 - There's a subfield of environmental economics called "stated preference" which does exactly this.
- An indirect approach infers WTP for Δq indirectly through private good markets.
 1. **Complements:** improved water quality in a lake increases the value of a fishing trip.
 2. **Substitutes:** demand for clean drinking water could be produced by bottled or filtered water or clean tap water.
 3. **Quality-differentiated products:** the price of private goods adjusts with changes in environmental quality. This is where hedonic models come from.
- This indirect approach is called **revealed preference** because it uses observed decisions for private goods to infer welfare changes for changes in environmental quality.

Complements

Complements

Setup

- Goal: Measure WTP for environmental quality increase (from q^0 to q^1)
- Recall: $WTP \equiv CV = E(p, u^0, q^0) - E(p, u^0, q^1)$
- We can define:

$$a = \int_{\bar{p}(q^1)}^{p^0} h(p, u^0, q^1) dp - \int_{\bar{p}(q^0)}^{p^0} h(p, u^0, q^0) dp$$

where:

- p^0 is the price for good x
 - $\bar{p}(q^0)$ is the "choke price", i.e., the price that would drive demand to zero at $q = q^0$
 - $\bar{p}(q^1)$ is the "choke price", i.e., the price that would drive demand to zero at $q = q^1$
- Intuition: Difference between total area under Hicksian demand curve (and above price) at new quality and total area under Hicksian demand curve (and above price) at old quality.

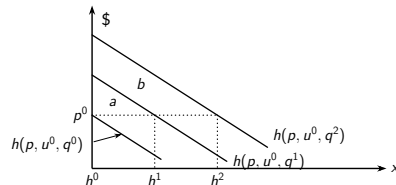
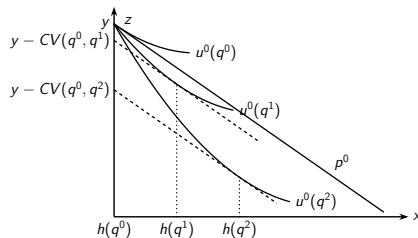
Complements

Weak complementarity

- a (from previous slide) = CV if the following holds:
 - $E(\bar{p}(q^1), u^0, q^1) - E(\bar{p}(q^0), u^0, q^0) = 0$
 - Intuition: If the private good is not purchased because its price exceeds the choke price, then changes in q do not affect the expenditure needed to reach u^0 .
 - In other words, q only has “**use value**.” Without consumption of a private good, q has no value.
 - *Example*: Trout in a stream have no value to me if I don't go fishing at that stream
 - *Violation*: I might care about habitat for polar bears, even if I never go to the Arctic to see them.

Weak Complementarity

- Suppose an individual's preference function is such that
 - (a) x is nonessential (i.e., $x = 0$ can be optimal)
 - (b) $\frac{\partial U(0, z, q)}{\partial q} = 0 \leftrightarrow U(0, z, q^1) = U(0, z, q^0)$
- Then x and q are **weak complements**.



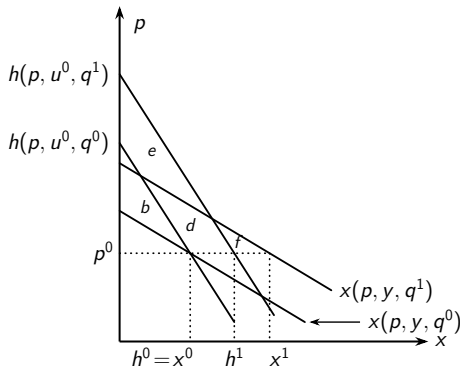
- Weak complementarity: every $u^0(q)$ meets the z -axis at y .
- Area $a = CV(q^0 \rightarrow q^1)$
- Area $a + b = CV(q^0 \rightarrow q^2)$

Turning to ordinary demand...

- With weak complementarity (i.e., q only has *use value*), we can measure WTP as area under Hicksian demand curves.
- But we only observe ordinary demand curves, which allow us to define a consumer surplus-like measure:

$$c = \int_{p^0}^{\bar{p}(q^1)} x(p, y, q^1) dp - \int_{p^0}^{\bar{p}(q^0)} x(p, y, q^0) dp$$

- Where $\bar{p}(\cdot)$ are choke prices for $x(\cdot)$ at different levels of q
- When is this useful for welfare analysis?
 - In general, c cannot be bounded by CV as before with price changes
 - Change in Hicksian demand and Marshallian demand w.r.t change in q differ by an income effect. (Similar argument for WTP vs. WTA)



- $CV(q^0 \rightarrow q^1) = d + e$.
- $c(q^0 \rightarrow q^1) = b + d + f$.
- In general $c \geq CV$.

Complements: The Willig Condition

- In Ch. 14, we explored conditions for when consumer surplus (c) would be a good approximation for WTP for changes in q .

$$C = \int_{q^0}^{q^1} \theta^q(p, y, q) dq = \int_{q^0}^{q^1} \frac{\partial V(p, y, q)/\partial q}{\partial V(p, y, q)/\partial y} dq$$

- Recall, θ^q is the ordinary demand function (or MWTP function) for q , so C is consumer surplus under a “hypothetical” ordinary demand function (if q were traded in markets).
- When will $c = C$? That is, when will consumer surplus (based on ordinary demand for x) equal the similar measure (C) for our hypothetical ordinary demand for q ?

The Willig Condition

If $V(p, y, q)$ is such that the income elasticity of demand for x is equal to the income elasticity of the ordinary inverse demand for q , or:

$$\eta \equiv \frac{\partial x(\cdot)}{\partial y} \frac{y}{x(\cdot)} = \frac{\partial \theta^q(\cdot)}{\partial y} \frac{y}{\theta^q(\cdot)} \equiv \zeta$$

- Then, $c = C$.

⇒ If Willig condition is satisfied, then integrating over price changes (with ordinary demand curves for x) is equivalent to integrating over quantity changes (with the MWTP function, θ^q , for q).

Complements

Incorporating q into price effects

- We can “repackage” the consumers’ problem by directly tying q and x together.
 - These assumptions can satisfy weak complementarity by definition.
 - Requires assumptions about how x and q are related...
- Goal: Set up the consumer’s problem so optimal choice of x is defined by **“quality-adjusted” prices**.
 - “Simple repackaging”:

$$\max_{x,z} U(x \cdot \delta(q), z) \text{ s.t. } y = px + z \implies x = \frac{g(p/\delta(q), y)}{\delta(q)}$$

- Or, “cross-product repackaging”:

$$\max_{x,z} U(x, z + x \cdot \delta(q)) \text{ s.t. } y = px + z \implies x = g(p - \delta(q), y)$$

$\implies$ Ordinary demand functions include a quality-adjusted price.

Substitutes

Substitutes

Model set-up

- General idea: x and q are intermediate inputs to producing some commodity H .
 - Consumer gets utility from consuming H , but not x or q directly.
 - E.g., H is water consumption, q is quality of drinking water supply and x is water filtration technology or bottled water purchases.
 - $H = f(x, q)$ where f is a household production function.
 - H can be produced with more x or more q (i.e., substitutes in production)
 - Define a cost function: $C(p, q, H) = \min_x \{px + \mu[H - f(x, q)]\}$
- Consumers then maximize: $\max_{x,z} U(H, z)$ s.t. $y = C(p, q, H)$, $H = f(x, q)$
 - Which leads to ordinary demand functions: $x(p, y, q)$ and $z(p, y, q)$
- Or, set up expenditure minimization: $E(p, u, q) = \min_{x,z} \{px + z$ s.t. $U(H, z) = u$, $H = f(x, q)\}$
 - Which leads to compensated demand functions: $h(p, u, q)$

Substitutes

WTP for (marginal) changes in q

- From expenditure minimization problem, we can express MWTP for small changes in q (holding H constant):

$$MWTP = -\frac{\partial E(p, u, q)}{\partial q} = -\frac{\partial C(p, q, H)}{\partial q} \implies -\frac{\partial C(p, q, H)}{\partial q} = p \frac{\partial f(\cdot)/\partial q}{\partial f(\cdot)/\partial x}$$

- Differentiating w.r.t q also provides: $-p \frac{\partial x(\cdot)}{\partial q} = \frac{\partial f(\cdot)/\partial q}{\partial f(\cdot)/\partial x}$

$\implies$ marginal changes in q can be defined by either the cost fn, production fn, or input demand fn for x .

Substitutes

WTP for (non-marginal) changes in q

- Define two new concepts:
- Actual savings in spending on x resulting from a change in q :

$$AS = C(p, q^0, H^0) - C(p, q^1, H^1) = p[x(p, y, q^0) - x(p, y, q^1)]$$

- Potential savings in spending on x following a change in q (holding H constant):

$$PS = C(p, q^0, H^0) - C(p, q^1, H^0)$$

- In general, these are not equivalent to either of our welfare measures previously.
- Can derive exact welfare measures by placing further restrictions on technology:
 - x is an essential input to $H \Rightarrow$ we can use $x(\cdot)$ to measure c , which approximates CV
 - $f(x, q)$ is separable in its inputs $\Rightarrow$ we can use $C(\cdot)$ to measure WTP directly
- Can place bounds on AS and PS to relate them to measures of CV

Substitutes

Recap

- Specific measures of welfare do not fall out of the model
 - Except in special cases where additional restrictions are put on $f(x, q)$
- This framework is often used to measure “averting behavior/defensive expenditures” in health/environment.
 - E.g., demand for bottled water in response to hurricanes.
 - Beatty, Timothy KM, Jay P. Shimshack, and Richard J. Volpe. “Disaster preparedness and disaster response: Evidence from sales of emergency supplies before and after hurricanes.” *Journal of the Association of Environmental and Resource Economists* 6, no. 4 (2019): 633–668.
<https://www.journals.uchicago.edu/doi/full/10.1086/703379>.
 - E.g., purchases/behavioral adjustments in response to air pollution.
 - Barwick, Panle Jia, Shanjun Li, Liguang Lin, and Eric Yongchen Zou. “From fog to smog: The value of pollution information.” *American Economic Review* 114, no. 5 (2024): 1338–1381.
<https://www.aeaweb.org/articles?id=10.1257/aer.20200956>.

Quality-differentiated goods

Quality-differentiated goods

- The complements and substitutes frameworks assume that changes in q do not affect the market price of x .
- With quality-differentiated goods, we can model q as a characteristic of market goods.
 - E.g., clean air or proximity to open space increases the value of a home
 - Or, higher wages intended to compensate for riskier jobs/occupations
 - Or, fuel efficiency in vehicles
- q is indirectly priced into the market good.
 - How do we think about differentiated goods in equilibrium?
 - How do we derive MWTP for q in these settings?

Quality-differentiated goods

The hedonic model

- A market good can be described by a J -dimensional vector of its characteristics, x , and the associated environmental quality, q .
 - Household preferences are defined over x and q , not the produced commodity.
 - Market price of the commodity is $P(x, q)$, which is the **hedonic price function**.
- Consumer's max: $\max U(x, q, z; s)$ s.t. $y = P(x, q) + z$
 - Now consumers choose q !
 - s denotes characteristics of the individual
- FOCs:

$$\frac{\partial U(\cdot)}{\partial x_j} = \lambda \frac{\partial P(x, q)}{\partial x_j} \quad \text{for } j = 1, \dots, J; \quad \frac{\partial U(\cdot)}{\partial q} = \lambda \frac{\partial P(x, q)}{\partial q}, \quad \frac{\partial U(\cdot)}{\partial z} = \lambda$$

- Combining these provides:

$$\frac{\partial U(\cdot)/\partial q}{\lambda} = \frac{\partial U(\cdot)/\partial q}{\partial U(\cdot)/\partial z} = \frac{\partial P(x, q)}{\partial q}$$

⇒ Individual chooses q to equate MRS of q for z (i.e., the MWTP for q) to the “implicit” marginal price of q .

Quality-differentiated goods

MWTP for q

- Let $\bar{u}$ represent a reference level of utility, such that $\bar{u} = U(x, q, z; s)$
 - Also, let $z = U^{-1}(x, q, \bar{u}; s)$
- Define a dollar amount b and attribute levels x and q so that $U(x, q, y - b; s) = \bar{u}$
 - This implicitly defines a bid function $b(x, q, y, s, \bar{u})$ that measures the maximum amount a consumer would pay for a commodity unit with attribute levels x and q holding utility fixed at $\bar{u}$. $\Rightarrow b(x, q, y, s, \bar{u}) \equiv y - U^{-1}(x, q, \bar{u}; s)$
- Implicitly differentiating U w.r.t. q provides:

$$0 = \frac{\partial U(\cdot)}{\partial q} - \frac{\partial U(\cdot)}{\partial z} \frac{\partial b(\cdot)}{\partial q} \Rightarrow \pi^q(x, q, s, \bar{u}) \equiv \frac{\partial b(x, q, y, s, \bar{u})}{\partial q} = \frac{\partial U(\cdot)/\partial q}{\partial U(\cdot)/\partial z}$$

- Where $\pi^q(\cdot)$ is the compensated MWTP function for q (i.e., holds utility constant)
- At the consumer's optimum, $\frac{\partial b(x, q, y, s, u^0)}{\partial q} = \frac{\partial P(x, q)}{\partial q}$...so, we want to use market data to estimate how $P(x, q)$ changes at different levels of q

Quality-differentiated goods

The hedonic model: Producers

- If q is a public good (or provided by nature/government), then $P(x, q)$ is determined by consumer preferences.
- If q is produced by firms (e.g., fuel economy of vehicles), then we need both consumers and producers.
- Firms produce the commodity with costs: $C(x, q; v)$, where v captures firm heterogeneity
- Firms maximize: $\max_{x, q} \Pi = P(x, q) - C(x, q; v)$

$$\Rightarrow \frac{\partial P(\cdot)}{\partial q} = \frac{\partial C(\cdot)}{\partial q}; \quad \frac{\partial P(\cdot)}{\partial x_j} = \frac{\partial C(\cdot)}{\partial x_j}, \quad j = 1, \dots, J$$

- Now, define o as the amount firms would accept to produce one unit of the commodity, achieving $\bar{\Pi}$, so that $\bar{\Pi} = o - C(x, q; v)$. This defines an offer function $o(x, q, \bar{\Pi}, v)$ which is firms' WTA.
 - Differentiating gives MWTa for q :

$$\frac{\partial o(x, q, \bar{\Pi}, v)}{\partial q} = \frac{\partial C(x, q; v)}{\partial q}$$

- which implies at optimum:

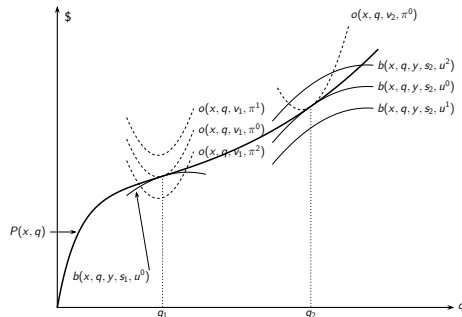
$$\frac{\partial o(x, q, \bar{\Pi}, v)}{\partial q} = \frac{\partial P(x, q)}{\partial q}$$

Quality-differentiated goods

The hedonic model: Equilibrium

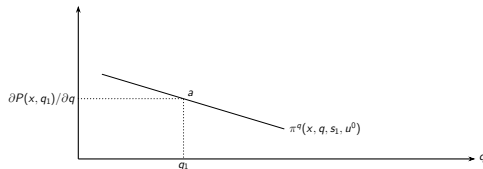
Top panel:

- Two consumers (s_1 and s_2); two firms (v_1 and v_2)
- Dashed lines: Firm offer curves based on different profit levels.
- Solid lines: Consumer bid curves based on different utility levels.
- **Thick bold line**: Equilibrium price function $P(x, q)$.



Bottom panel:

- A single point is identified on consumer 1's MWTP for q function for a given quantity (q_1).
- Cannot identify full MWTP function without more assumptions (focus of Ch. 18).



Key takeaways

- Measuring welfare for environmental goods depends on the context and requires theoretical assumptions/restrictions.
 - If x and q are **complementary**, then demand for x can tell us about welfare impacts of changes in q under certain conditions (i.e., weak complementarity, or when q only has “use value”)
 - If x and q are **substitutes** in producing a commodity, welfare measures can be obtained with restrictions on the productive relationship between x and q
 - If q is a **characteristic of x that influences its price**, then the hedonic price function tells us about consumers' MWTP for q
- Theoretically, calculating welfare for changes in environmental quality is relatively simple. These approaches are all geared towards what we can observe empirically.

For next class...

- **Mon Oct 5:** Fall break – no class.
- **Wed Oct 7:** Attend [SWEEP](#) in lieu of class.
- Readings and deadlines: [course website](#).