

ECON-7102: Environmental Economics I

Class 10: Welfare analysis

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Fall 2026

Today's agenda

1. Questions?
2. Rapid-fire research sketches
3. Today:
 - Welfare analysis (for price changes and *environmental quality* changes)
4. For next class...

Motivation

- Up until now, we've been focusing on modeling a simplified economy with externalities.
- Generally speaking, optimal policy equalizes marginal costs of reducing pollution with the marginal benefits of doing so.
 - Although, there are exceptions to this rule (e.g., pre-existing distortions).
- Incentive-based policies tend to have a competitive edge over alternative policies.
- So, just set τ equal to marginal external damages. (Or, set a cap on emissions and let the market figure it out).
- But...how do we measure marginal damages?
 - That's the focus of the next section of the course: *understanding what people are willing to pay to avoid environmental damages (or, similarly, what they'd be willing to accept for a degraded environment).*

New model set-up

- Focusing on consumers
 - Marginal damages are based entirely on the externality borne by consumers
- Utility is now $U(x, z, q)$
 - x is a vector of private goods (sold at prices p_j)
 - z is a numeraire good (with price normalized to 1)
 - q is an environmental public good

$$\max_{x,z} \{ U(x, z, q) + \lambda[y - z - p'x] \}$$

- $\partial U / \partial x_j = \lambda p_j$ (for $j = 1, \dots, J$)
 - $\partial U / \partial z = \lambda$
 - λ is the marginal utility of income.
- FOCs define **ordinary** demand functions: $x_j(p, y, q)$, $\lambda(p, q, y)$, and optimal z .
 - Plugging these into $U(\cdot)$ leads to **indirect utility function**: $V(p, y, q)$
 - Note: income & prices do not generate utility directly, only **indirectly** through the consumption of goods.

New model set-up

Can also express consumer's problem via expenditure minimization

$$\min_{x,z} \{p'x + z + \mu[\bar{u} - U(x, q, z)]\}$$

- $\bar{u}$ is reference level of utility
 - $\partial U / \partial x_j = p_j / \mu$ (for $j = 1, \dots, J$)
 - $\partial U / \partial z = 1 / \mu$
 - $U(x, q, z) = \bar{u}$
- ...yielding **compensated** demand functions $h_j(p, \bar{u}, q)$ for each x_j and $h_z(p, \bar{u}, q)$ for z , which depend on levels of *utility*, not income.
 - Compensated demand functions cannot be estimated/observed.
- Plug into expenditure minimization to obtain: $E(p, \bar{u}, q)$, which is the expenditure needed to obtain $\bar{u}$.

Duality between utility max. and expenditure min.

- If u^0 is maximum utility from utility maximization problem,
 - $E(p, u^0, q)$ is expenditure needed to obtain $u^0 \implies E(p, u^0, q) = y$
- Then, $x_j(p, E(p, u^0, q), q) \equiv h_j(p, u^0, q) \forall j$.
 $\implies$ compensated and ordinary demand functions are equal at point of consumption.
- Differentiating both sides w.r.t. p_j leads to the **Slutsky equation**:

$$\frac{\partial x_j(p, y, q)}{\partial p_j} = \underbrace{\frac{\partial h_j(p, u^0, q)}{\partial p_j}}_{\text{Price effect}} - \underbrace{\frac{\partial x_j(p, y, q)}{\partial y} \times \frac{\partial E(p, u^0, q)}{\partial p_j}}_{\text{Income effect}}$$

where the last term: $\frac{\partial E(p, u^0, q)}{\partial p_j} = x_j(p, y, q)$

- Compensated/ordinary demand functions differ by an income effect.
 - If income effect is zero, then $x_j = h_j$.

Compensated vs. ordinary demand

Slutsky equation

$$\frac{\partial x_j(p, y, q)}{\partial p_j} = \underbrace{\frac{\partial h_j(p, u^0, q)}{\partial p_j}}_{\text{Price effect}} - \underbrace{\frac{\partial x_j(p, y, q)}{\partial y}}_{\text{Income effect}} \times x_j(p, y, q)$$

- $x_j(p, y, q)$ = ordinary demand = Marshallian demand = demand for good j holding income constant.
- $h_j(p, u^0, q)$ = compensated demand = Hicksian demand = demand for good j holding utility constant.
- *Intuition:* a price increase makes you poorer at fixed income, and ordinary demand carries that loss. Compensated demand hands back the income needed to hold utility fixed, so only the substitution remains.



This is important enough to be inscribed in the SOE hallway.

Welfare measures for price changes

Compensating variation (CV)

- Generally, we want to know how welfare changes when price/quantity/quality changes.
- We can't observe utility, so we focus on how much individuals are *willing to pay* or *willing to accept* to experience the change in prices.
 - We also cannot observe Hicksian/compensated demand.

Compensating variation (CV)

Given a price decrease (increase), from p^0 to p^1 , CV is the amount of money that would need to be taken from (given to) a person to restore the original utility level.

$$V(p^0, y, q) = V(p^1, y - CV, q)$$

- Reference point is utility at original price levels.
- $CV > 0$ for $\downarrow$ price (WTP for $\downarrow p$); $CV < 0$ for $\uparrow$ price (WTA for $\uparrow p$).
- $CV = E(p^0, u^0, q) - E(p^1, u^0, q) = y - E(p^1, u^0, q)$
- $CV = \int_{p_j^1}^{p_j^0} h_j(p_j, p_{-j}, u^0, q) dp_j$

Welfare measures for price changes

Equivalent variation (EV)

Equivalent variation (EV)

For a price decrease (increase), EV is the payment that would need to be given (taken) to the individual to reach the same utility without the price change.

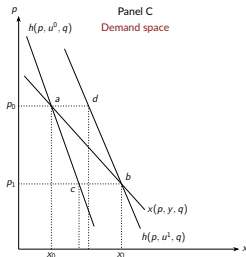
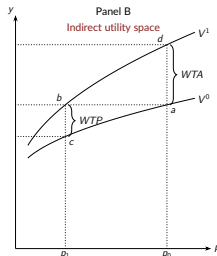
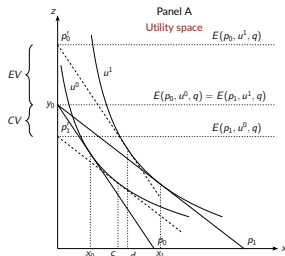
$$V(p^1, y, q) = V(p^0, y + EV, q)$$

- Reference point is utility at new price levels.
- $EV > 0$ for $\downarrow$ price (WTA to forgo $\downarrow p$), $EV < 0$ for $\uparrow$ price (WTP to avoid $\uparrow p$).
- $EV = E(p^0, u^1, q) - E(p^1, u^1, q) = E(p^0, u^1, q) - y$
- $EV = \int_{p_j^1}^{p_j^0} h_j(p_j, p_{-j}, u^1, q) dp_j$

Welfare measures for price changes

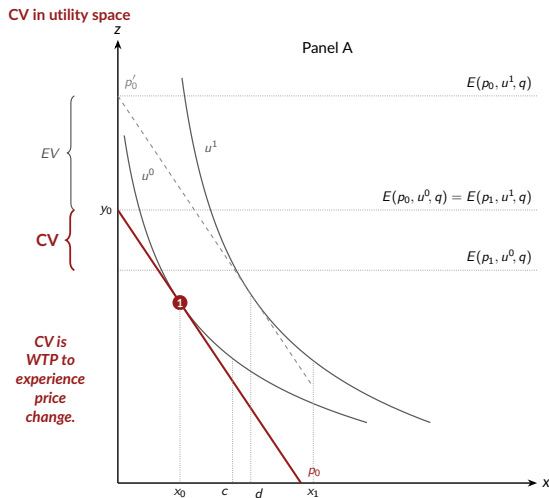
Prices fall from p_0 to p_1 : three views of one change

- **A.** The budget line rotates out and the consumer moves to x_1 . CV takes income away until u^0 is restored at the new price ratio, point c . EV adds income until u^1 is reached at the old price ratio, point d .
- **B.** The same two gaps, read vertically. CV is WTP for the price cut, EV is WTA to forgo it. Reversed for a price increase.
- **C.** The same two gaps, read as areas. $CV = p_0 a c p_1$ and $EV = p_0 d b p_1$. Only a and b are observed.



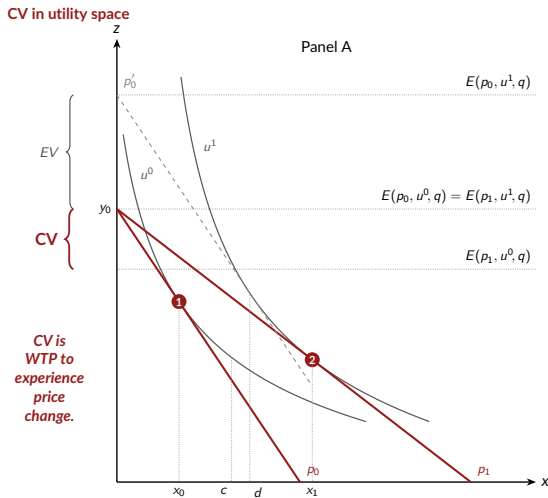
Welfare measures for price changes

Compensating variation (CV), graphically for $\downarrow$ price



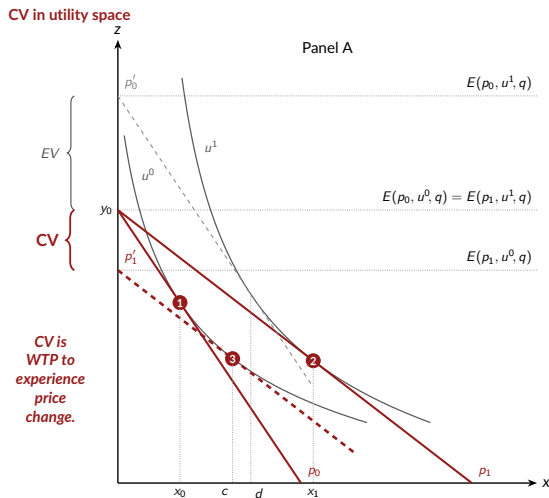
Welfare measures for price changes

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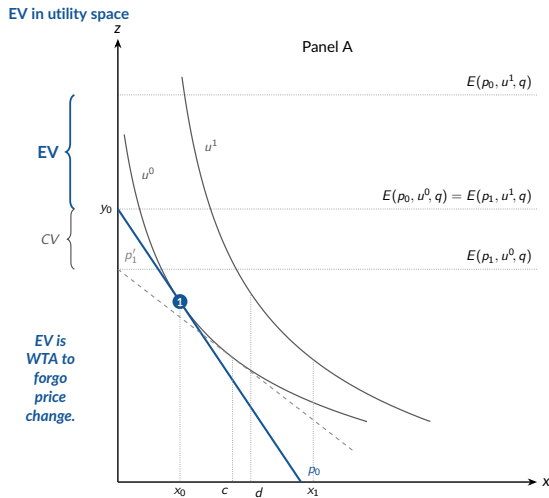
Welfare measures for price changes

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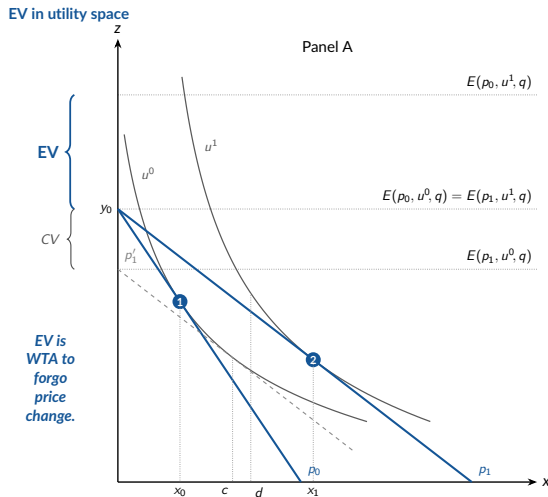
Welfare measures for price changes

Equivalent variation (EV), graphically for $\downarrow$ price



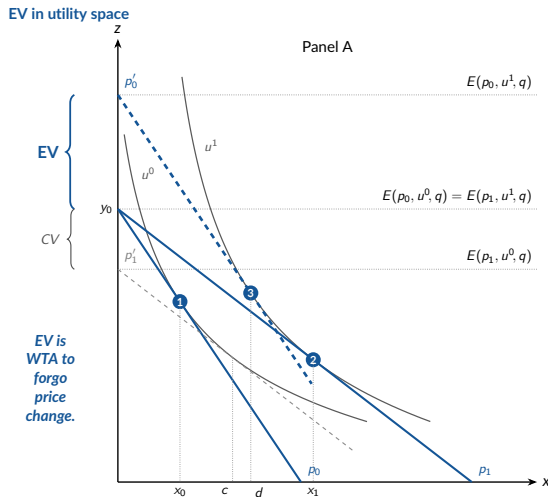
Welfare measures for price changes

Equivalent variation (EV), graphically for $\downarrow$ price



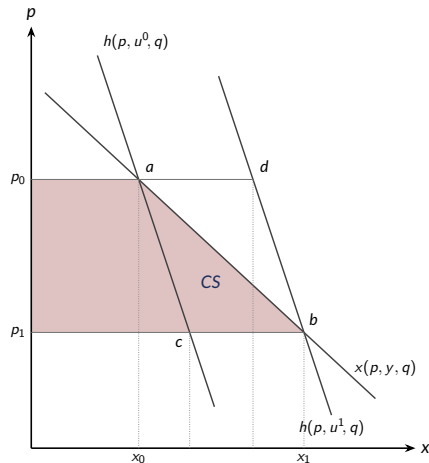
Welfare measures for price changes

Equivalent variation (EV), graphically for $\downarrow$ price



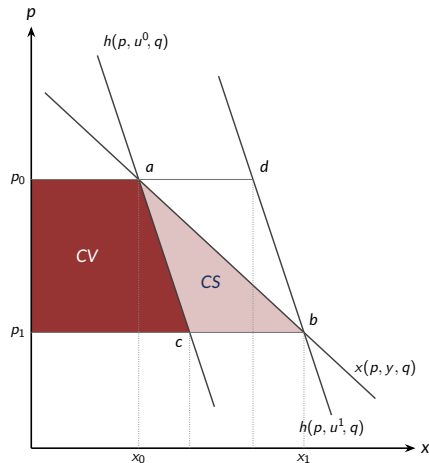
Wait, what about consumer surplus!?

- CS from price change is area under Marshallian (or ordinary) demand curve between the old/new prices.
 - Note: *not* a measure of WTP or WTA.
- Often used as an approximation because we can estimate $x(p, y, q)$



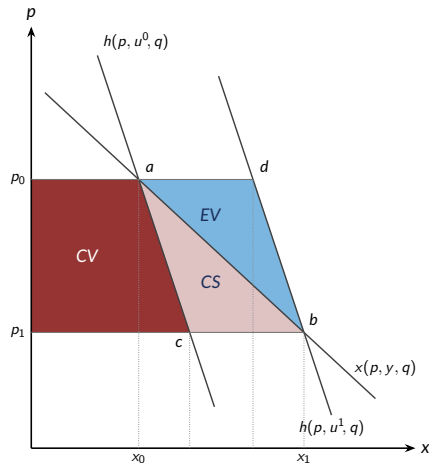
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- CV is area p_0acp_1 (in darker red)



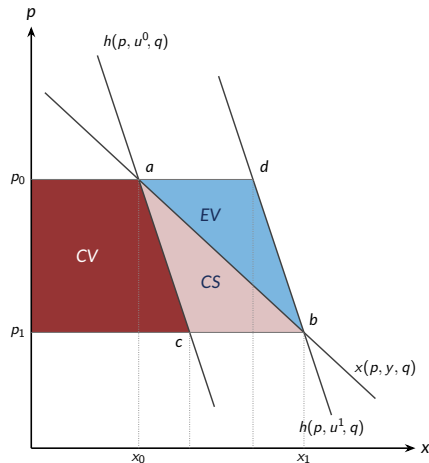
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- CV is area $p_0 a c p_1$ (in darker red)
- EV is area $p_0 d b p_1$ (in red + blue)



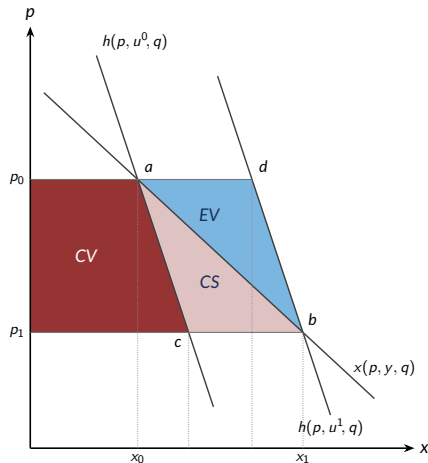
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 - Note: *not* a measure of WTP or WTA.
- Often used as an approximation because we can estimate $x(p, y, q)$
- CV is area p_0acp_1 (in darker red)
- EV is area p_0dbp_1 (in red + blue)
- In general, we can state that $CV < CS < EV$.



Wait, what about consumer surplus!?

- How far off from CV or EV is CS ?
 - If no income effects are present, $CS = EV = CV$.
 - Also if marginal utility of income is constant w.r.t. p_j , then CS has a quantitative welfare interpretation.
 - In general, for price changes, CS works well as an approximation.
 - Lots of papers written about this, including classic Willig (1976) paper
 - Same logic implies that WTP and WTA are similar in magnitude.
- What can economists do, practically?
 1. Estimate ordinary demand functions w.r.t. price change.
 - These need to satisfy “regularity” conditions consistent with theory.
 2. “Integrate back” to obtain expenditure and indirect utility functions.
 3. Calculate CV or EV based on (functions of) estimated parameters.



Welfare measures *for environmental quality changes*

- Recall, this is an environmental economics course.
- Environmental goods are typically not traded in markets.
 - How much are you willing to pay for cleaner air?
 - How much would you need to be compensated for more contaminants in your drinking water?
 - How much would you be willing to pay to avoid the Deepwater Horizon oil spill?
- **How can we evaluate changes in welfare when the quality of environmental goods changes?**

Welfare measures for *environmental quality* changes

Compensating variation (CV)

Compensating variation (CV) for env. quality changes

$$V(p, y, q^0) = V(p, y - CV, q^1)$$

- Reference point is utility at original quantity levels.
- WTP for $\uparrow q$. Person is not entitled to improvement.
- $CV > 0$ for a quantity increase, $CV < 0$ for quantity decrease.

$$CV = E(p, u^0, q^0) - E(p, u^0, q^1) = y - E(p, u^0, q^1)$$

$$CV = \int_{q^0}^{q^1} \pi^q(p, u^0, q) dq = \int_{q^0}^{q^1} -\partial E(p, u^0, q) / \partial q dq = E(p, u^0, q^0) - E(p, u^0, q^1)$$

Note: Define $\pi^q(p, u, q) = -\partial E(p, u, q) / \partial q$ as the compensated inverse demand for q . This also defines the MWTP function for q .

Welfare measures for *quantity (quality)* changes

Equivalent variation (EV)

Equivalent variation (EV) for env. quality changes

$$V(p, y, q^1) = V(p, y + EV, q^0)$$

- Reference point is utility at new quantity levels.
- WTA to forgo change in q . Person is entitled to improvement.
- $EV > 0$ for a quantity increase, $EV < 0$ for quantity decrease.

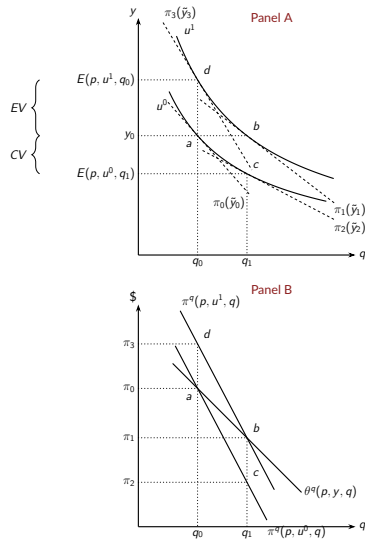
$$EV = E(p, u^1, q^0) - E(p, u^1, q^1) = E(p, u^1, q^0) - y$$

$$EV = \int_{q^0}^{q^1} \pi^q(p, u^1, q) dq = \int_{q^0}^{q^1} -\partial E(p, u^1, q) / \partial q dq = E(p, u^1, q^0) - E(p, u^1, q^1)$$

Note: Define $\pi^q(p, u, q) = -\partial E(p, u, q) / \partial q$ as the compensated inverse demand for q . This also defines the MWTP function for q .

Welfare measures for *environmental quality* changes

- Quantity changes work virtually identically to price changes except we now define “virtual” prices π which trace out marginal WTP at different levels of q .
 - If q were traded in markets, the individual would be willing to pay π for an additional unit.
- For price changes, points a and b are observable (e.g., demand at two prices).
 - This is not the case for quantity changes since q is not traded in markets.



Welfare measures for *environmental quality* changes

○ WTP vs. WTA...

- With price changes, WTP and WTA will be similar (especially with small income effects). $\implies$ area $adbc$ is small.

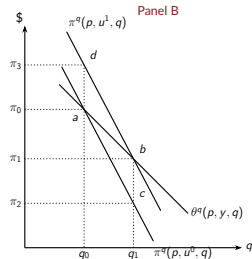
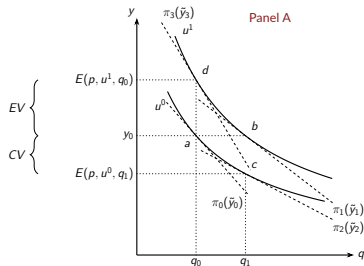
○ For environmental goods, $WTP \neq WTA$ in general, and the difference could be large.

- WTP is limited by income, WTA is unbounded.
- Meta-analysis suggests that in dozens of published studies WTA is 7x greater than WTP (Horowitz & McConnell, 2002)

○ Hanemann (1991) shows that divergence in WTP and WTA depends on degree of substitutability of q .

- Income effect can be decomposed into pure income effect + pure substitution effect
- If q can be easily replaced with market goods (i.e., $\pi^q(\cdot)$ is elastic), then income effect is small. If inelastic, then income effect is large.

$\implies$ **CS is a potentially poor approximation of WTP/WTA. But, more fundamentally, we can't use CS because we can't estimate ordinary demand curves for q directly.**



Key takeaways on applied welfare theory (so far)

- For price changes, we can construct measures of CV/EV (or WTP/WTB) based on standard theory.
 - Importantly, consumer surplus will likely be a good approximation of both CV and EV .
 - This allows us to use ordinary demand curves to approximate welfare metrics.
 - But, we can also “integrate back” from ordinary demand equations to calculate welfare changes exactly.
- For quantity changes (or changes in the amount of environmental quality provided), welfare is trickier for two reasons:
 - We can't estimate ordinary demand curves for environmental quality (because it's not traded in markets)
 - WTP and WTB can diverge substantially depending on the degree of substitutability between q and market goods.
- Where does that leave us?
 - Next lecture: using relationship between market and non-market goods to infer changes in welfare. This is the premise behind revealed preference measures of valuation, which we will spend a few weeks on.

For next class...

Class 10 – Valuation I (Revealed preference)

- ☐ P&R (Ch. 15)
- ☐ Hanemann, W. Michael. "Willingness to pay and willingness to accept: How much can they differ?" *American Economic Review*, 1991.
- ☐ Shogren, Jason F., Seung Y. Shin, Dermot J. Hayes, and James B. Kliebenstein. "Resolving differences in willingness to pay and willingness to accept." *American Economic Review*, 1994.

Class 11 – Valuation II (Property value models)

- ☐ P&R (Ch. 18) – Focus on 18.1 and 18.2.
- ☐ Chay, Kenneth Y., and Michael Greenstone. "Does air quality matter? Evidence from the housing market." *Journal of Political Economy*, 2005.
- ☐ Banzhaf, H. Spencer, and Randall P. Walsh. "Do people vote with their feet? An empirical test of Tiebout." *American Economic Review*, 2008.
- ☐ Christensen, Peter, and Christopher Timmins. "Sorting or steering: The effects of housing discrimination on neighborhood choice." *Journal of Political Economy*, 2022.

- For Wed: Ch. 15 (P&R).